城市交通研究院
Urban Mobility Institute

A New Formulation and an Exact Solution Approach for the Traveling Salesman Problem with a Drone Station

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Background

- **Last Mile Delivery Challenge:**
  - Most **inefficient** and **costly** part of logistics;
  - The Trend of **Urbanlization**: increased levels of **road congestion**;
  - **Aging populations**, **Online shopping**: increased demand for **home deliveries**.
- **Complementary Operation Characteristics of truck and drone:**

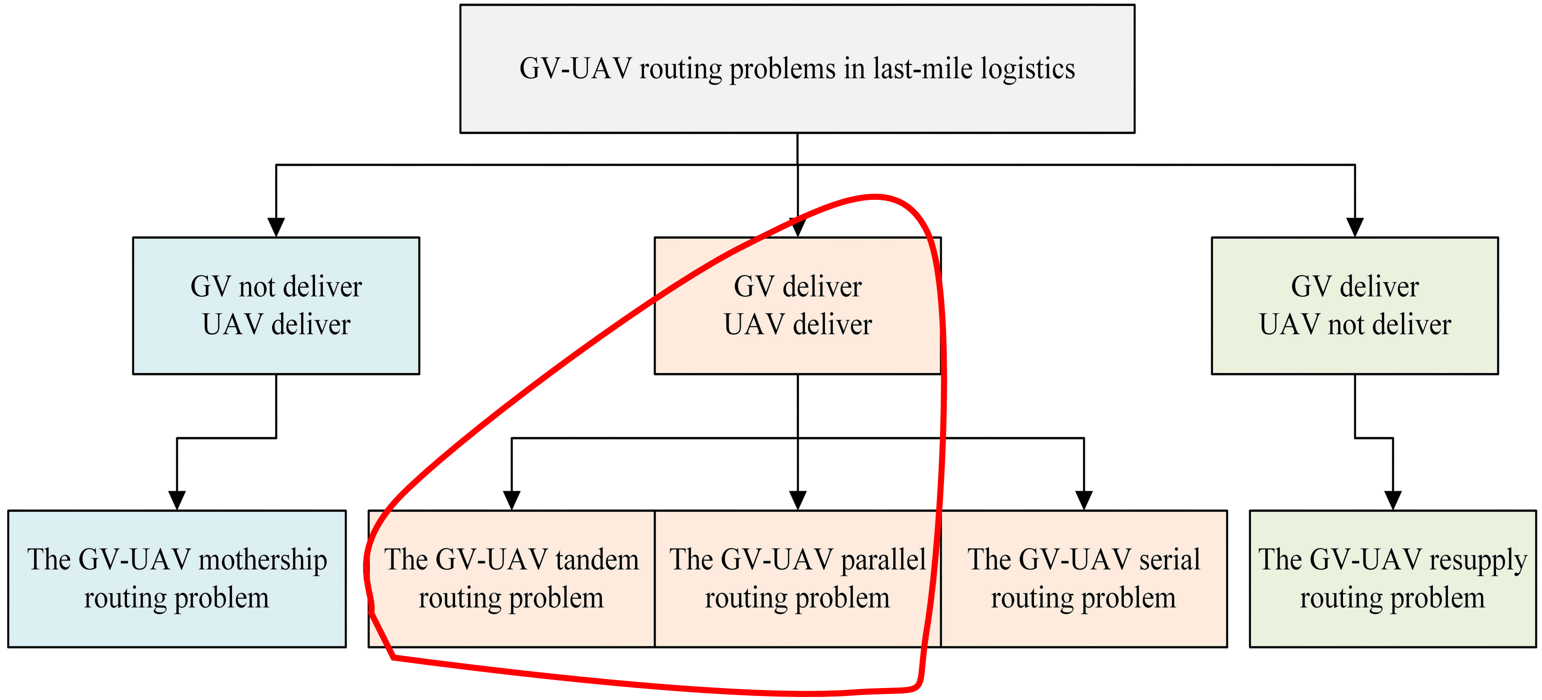
	Speed	Weight	Capacity	Range	Environmental impact	Relia
Drone (UAV)	High	Light	One	Short	Little	Lo
Truck (GV)	Low	Heavy	Many	Long	Big	Hi

- **Research Opportunity:**
  - The Ground-Vehicle and Unmanned-Aerial-Vehicle Routing Problem (GV-UAV-RP);
  - NP-hard VRP + **Cooperation** between GVs and UAVs;
  - Exact solution techniques for GV-UAV-RPs are rarely investigated.

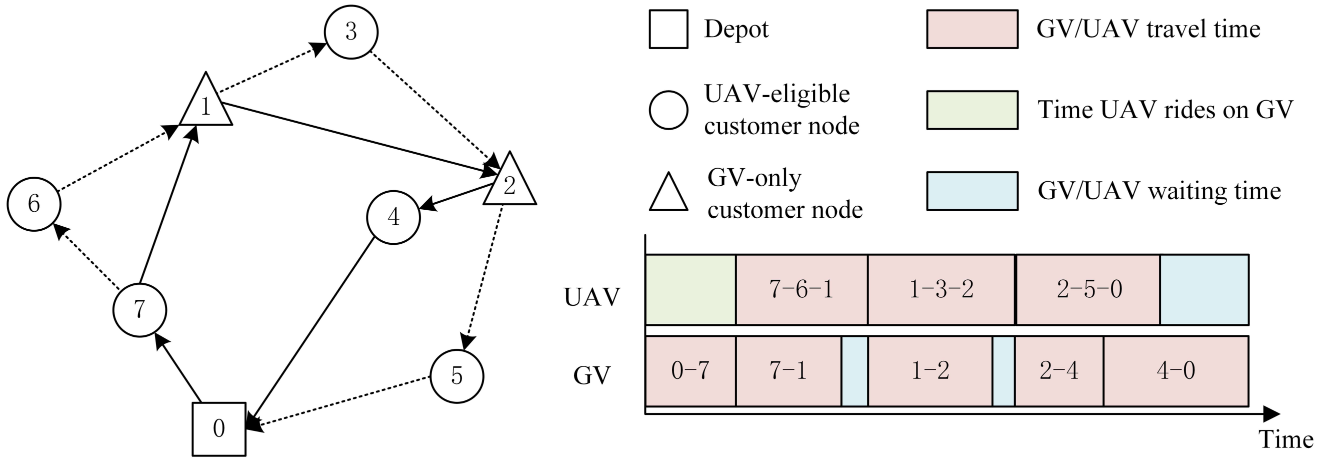
Contents

GV-UAV-RPs in last-mile logistics

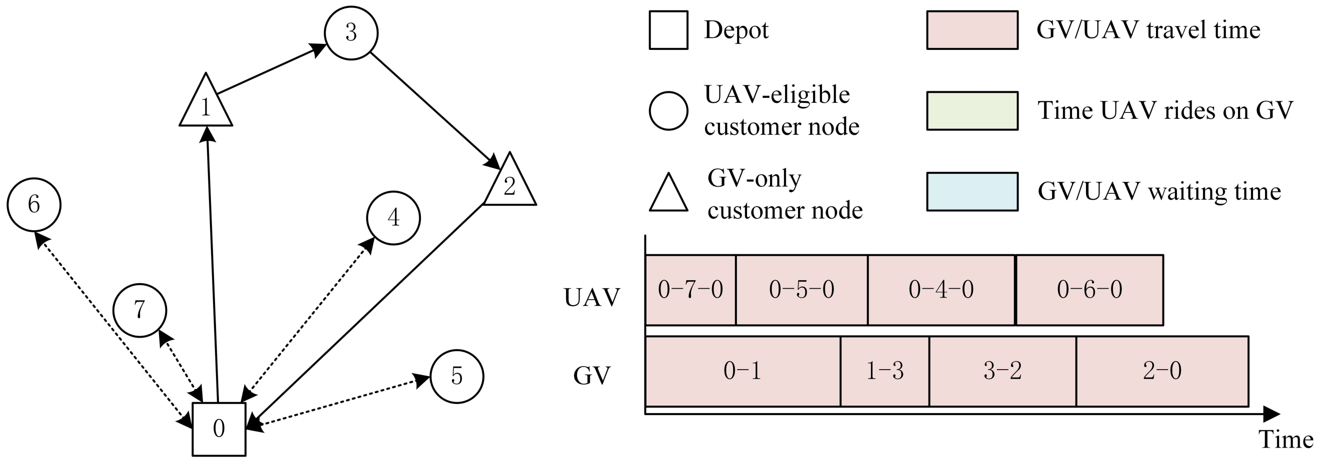
- In general, the GV-UAV routing problems consist of **three interdependent tasks**:
  - GV(s) routing, UAV(s) routing and **the cooperation between GV(s) and UAV(s)**
  - The **primary source of their complexity** compared to traditional VRPs.
  - A common form of cooperation is customer assignment, i.e., assigning customers to GVs respectively. Cooperation can also occur in the movement of GVs and UAVs, as UAVs can ride on t



Cooperation in tandem and parallel mod



- **Tandem mode**:
  - GVs carry UAVs and dispat
    customer nodes when making
  - UAV-eligible customers can
    to either a GV or a UAV;
  - Attract more attention c
    intriguing concept and
    complexity.



- **Parallel mode**:
  - UAV-eligible customers can
    to either a GV or a UAV.
  - Receives less attention
    apparent practical applicati
  - More effective when cus
    located near the depot.

Problem description of the TSP-DS

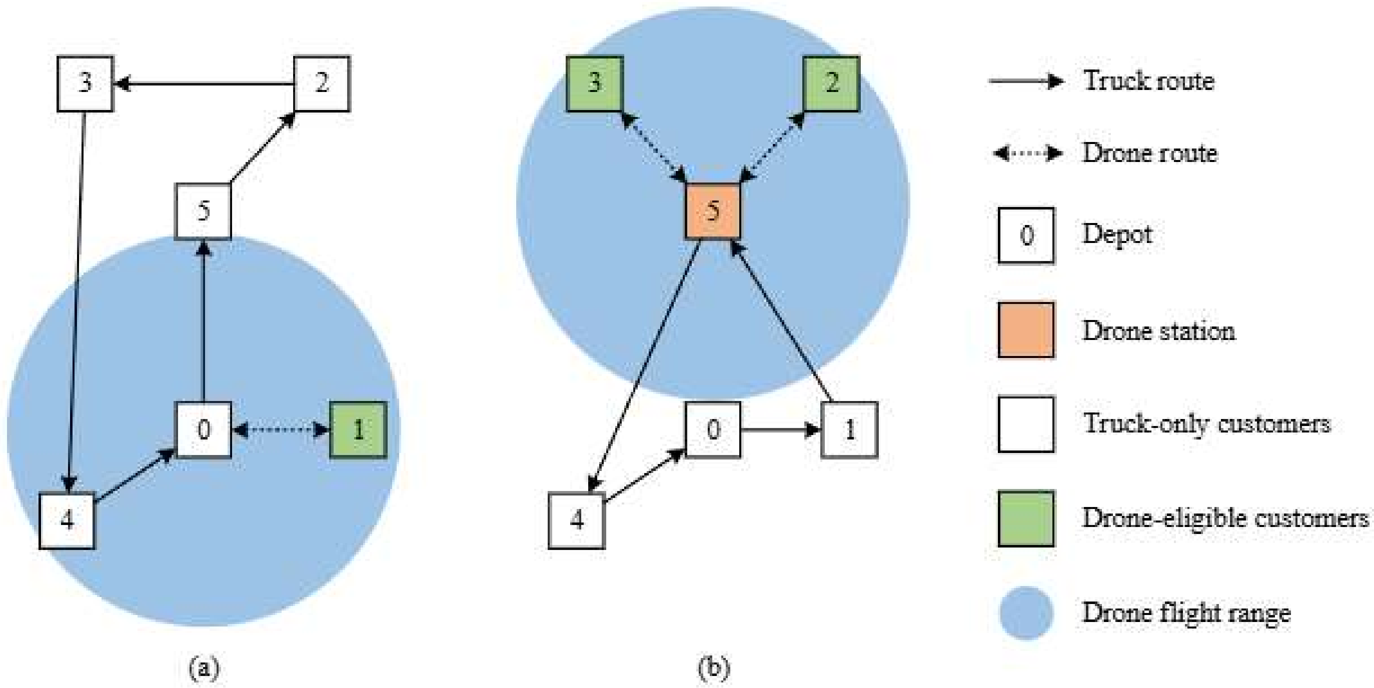


Figure 1: Comparison between the (a) PDSTSP and (b)TSP-DS.

- **PDSTSP**
  - **P**arallel **D**rone **S**cheduling Salesman **P**roblem;
  - Murray and Chu (2015);
  - Limited by depot location.
- **Drone station**
  - Overcome the limitation of
  - Activate by truck (supply p the drone station);
  - Support drone delivery;
  - Kim and Moon (2019);
  - TSP-DS.

A compact MILP formulation

- **Objective: Minimize the delivery makespan.**

Type	Notation	Definition
Set	N	Set of all nodes, $N = \{0, 1, ..., c+1\} \cup \{s\}$. $0, c+1$ denote the depot node and s denote the drone station node.
	N^-	Set of nodes from which the truck may depart, $N^- = \{0, 1, ..., c\} \cup \{s\}$.
	N^+	Set of nodes to which the truck may visit, $N^+ = \{1, 2, ..., c+1\} \cup \{s\}$.
	C	Set of customers, $C = \{1, 2, ..., c\} \cup \{s\}$. The drone station can be considered as a truck-only customer node.
	C'	Set of customers that can only be served by the truck, $s \in C'$, $C' \subset C$.
	C''	Set of customers that are eligible to be served by drones, $C' \cup C'' = C$.
	V	Set of drones, $V = \{1, 2, ..., \|V\|\}$.
Parameter	$\tau_{i,j}$	Truck travel-time from node $i \in N^-$ to node $j \in N^+$, $i \neq j$.
	$\tau'_{i,j}$	Drone travel-time from node $i \in C$ to node $j \in C$, $i \neq j$.
	M	A sufficiently large positive value.
Variable	$x_{i,j}$	Binary variables. $x_{i,j} = 1$, if the truck travels from node $i \in N^-$ to node $j \in N^+$, $i \neq j$; 0, otherwise.
	y_i^v	Binary variables. $y_i^v = 1$, if drone $v \in V$ serves customer $i \in C''$; 0, otherwise.
	a_i	Arrival time of the truck at node $i \in N$, if node $i \in N$ is in the truck path.
	T	Delivery completion time (makespan).

$$(P)\ \min T$$

s.t.

$$\sum_{\substack{j \in N^+ \\ j \neq i}} x_{i,j} = 1,\ \forall i \in C'$$

$$\sum_{\substack{j \in N^+ \\ j \neq i}} x_{i,j} + \sum_{v \in V} y_i^v = 1,\ \forall i \in C''$$

Truck-only custo… served by the … others could b… either the truck o…

$$\sum_{\substack{j \in N^+ \\ j \neq i}} x_{i,j} = \sum_{\substack{j \in N^- \\ j \neq i}} x_{j,i},\ \forall i \in C$$

$$a_i + \tau_{i,j} \le a_j + M(1 - x_{i,j}),\ \forall i \in N^-, j \in N^+, i \neq j$$

$$\sum_{i \in N^+} x_{0,i} = 1,\ \sum_{i \in N^-} x_{i,c+1} = 1$$

$$\sum_{\substack{i \in N^+ \\ i \neq s}} x_{s,i} = 1,\ \sum_{\substack{i \in N^- \\ i \neq s}} x_{i,s} = 1$$

Truck needs to… drone station.

$$T \ge a_{c+1}$$

$$T \ge a_s + \sum_{i \in C''} (\tau'_{s,i} + \tau'_{i,s}) y_i^v,\ \forall v \in V$$

Truck's mak…

Drones' mak…

$$x_{i,j} \in \{0, 1\},\ \forall i \in N^-, j \in N^+, i \neq j$$

$$y_i^v \in \{0, 1\},\ \forall i \in C'', v \in V$$

$$a_i \in R^+, \forall i \in N$$

$$T \in R^+$$

An improved MIP formulation

- **Motivation:** The big-M in the MTZ subtour elimination constraints result in very weak linear relaxati

$$\min \ T$$

$$\text{s.t.} \sum_{\substack{i\in N^-\\ i\neq j}} x_{i,j} + \sum_{\substack{v\in V\\ i\in C''}} y_i^v = 1, \ \forall j \in C$$

$$x_{i,j}^s \le x_{i,j}, \ \forall i \in N^-, j \in N^+, i \neq j$$

$$\sum_{\substack{j\in N^+\\ j\neq i\\ i\neq c+1}} x_{i,j}^s - \sum_{\substack{j\in N^-\\ j\neq i\\ i\neq 0}} x_{j,i}^s = \begin{cases} 1, & \text{if } i = 0 \\ -1, & \text{if } i = s \\ 0, & \text{otherwise.} \end{cases} \quad \forall i \in N$$

$$T \ge \sum_{i\in N^-} \sum_{\substack{j\in N^+\\ j\neq i}} \tau_{i,j} x_{i,j}^s + \sum_{i\in C''} (\tau'_{s,i} + \tau'_{i,s}) y_i^v, \ \forall v \in V$$

Introduce a set of auxiliary **continuous** variables to represent the activation time of the drone station.

$$T \ge \sum_{i\in N^-} \sum_{\substack{j\in N^+\\ j\neq i}} \tau_{i,j} x_{i,j}$$

$$\sum_{j\in N^+} x_{0,j} = 1$$

$$\sum_{i\in N^-} x_{i,c+1} = 1$$

$$\sum_{\substack{i\in N^-\\ i\neq j}} x_{i,j} = \sum_{\substack{i\in N^+\\ i\neq j}} x_{j,i}, \ \forall j \in C$$

Adopt the stronger GCS subtour elimination constraints.

$$\sum_{i\in S} \sum_{j\notin S} x_{i,j} \ge \sum_{i\in N^+\setminus\{k\}} x_{k,i}, \ \forall k \in S, S \subset N^-, |S| \ge 2$$

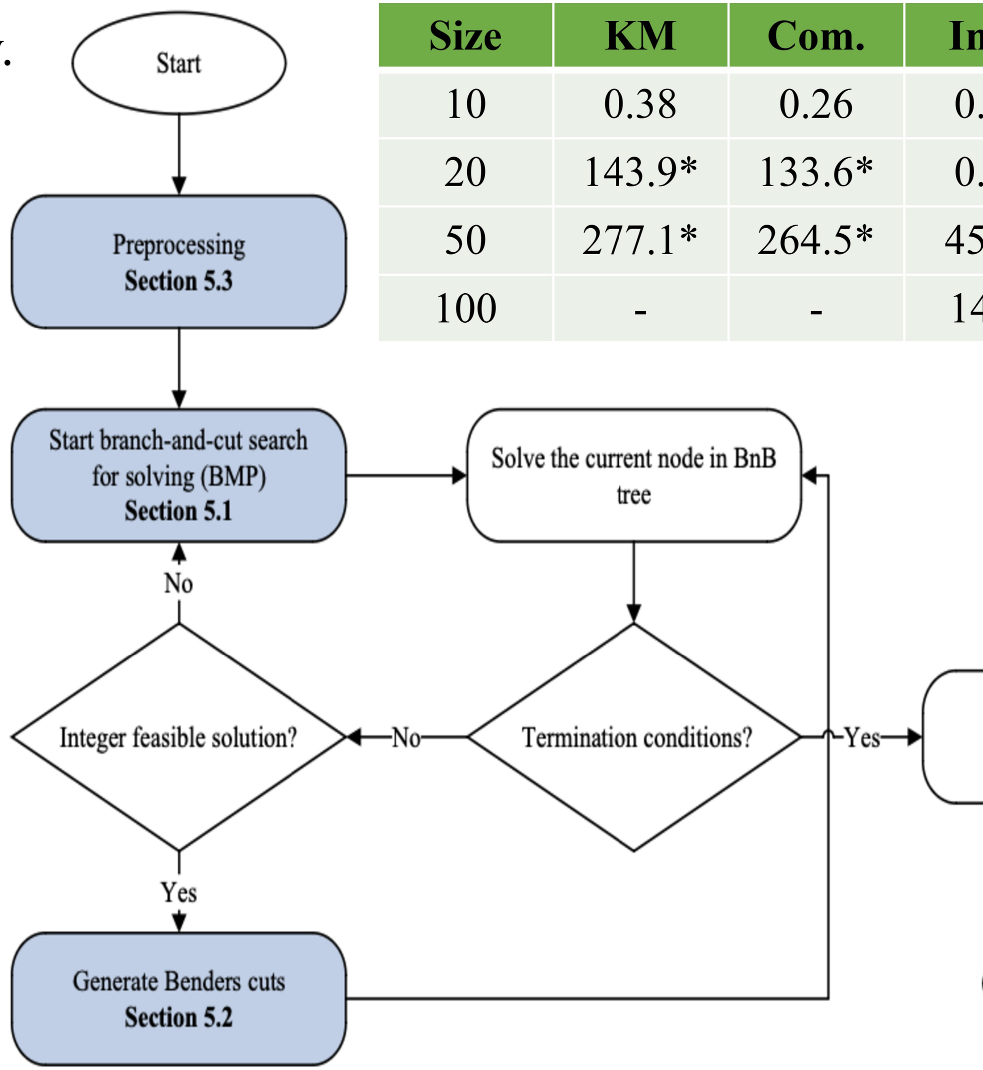
Algorithm 1 (Separation of GCS for Subto

1: $\mathbf{x}^* \leftarrow$ solution of the (BMP) at the node
2: $\epsilon \leftarrow 0.8$
3: Construct graph $G(N_{st}, A^*)$, wher $A|x_{ij}^* > 0$ or $x_{ji}^* > 0\}$
4: $\mathcal{S} \leftarrow \{S \subseteq N_s | S$ is a strongly connec component on $G\}$ ▷ Depth-firs $G(N_{st}, A^*)$
5: $\mathcal{C} \leftarrow \emptyset$
6: **for** $S \in \mathcal{S}$ **do**
7: **for** $k \in S$ **do**
8: $v \leftarrow \sum_{(i,j)\in\delta^+(\{k\})} x_{ij}^* - \sum_{(i,j)\in\delta^+}$
9: **if** $v \ge \epsilon$ **then**
10: $\mathcal{C} \leftarrow \mathcal{C} \cup \{(v, S, k)\}$
11: **end if**
12: **end for**
13: **end for**
14: **return** $\mathcal{C}$

Implementation c to Kang and Lee

Algorithm framework

- **Idea:** Separate truck routing and drone delivery.
- **Key components:**
    - A procedure for generating Benders cuts;
    - Preprocessing to speed up convergence.
- **Features:**
    - Logic-based Benders Decomposition;
    - Only one BnB tree.
- **Higher computational efficiency:**
    - Its advantage becomes more pronounced as the problem size grows.
- **Handle large scale instances well:**
    - Handle all instances with less than 150 customers;
    - Find global optimum for several instances with more than 200 customers



Size	KM	Com.	In
10	0.38	0.26	0.
20	143.9*	133.6*	0.
50	277.1*	264.5*	45
100	-	-	14

Benders master problem

$(BMP)\ \min a_s + W$ W denotes the delivery time after the truck activated the drone station.

$$\text{s.t.} \sum_{\substack{j \in N^+ \\ j \neq i}} x_{i,j} = 1, \forall i \in C'$$

$$\sum_{\substack{j \in N^+ \\ j \neq i}} x_{i,j} = z_i, \forall i \in C''$$

Auxiliary variable z connects the BMP and BSP.

$$\sum_{\substack{j \in N^+ \\ j \neq i}} x_{i,j} = \sum_{\substack{j \in N^- \\ j \neq i}} x_{j,i}, \forall i \in C$$

Truck routing

$$\sum_{i \in S} \sum_{j \notin S} x_{i,j} \geq \sum_{i \in N^+ \setminus \{k\}} x_{k,i}, \forall k \in S, S \subset N^-, |S| \geq 2$$

$$\sum_{i \in N^+} x_{0,i} = 1, \sum_{i \in N^-} x_{i,c+1} = 1$$

$$\sum_{\substack{i \in N^+ \\ i \neq s}} x_{s,i} = 1, \sum_{\substack{i \in N^- \\ i \neq s}} x_{i,s} = 1$$

$$W \geq \sum_{i \in N^-} \sum_{\substack{j \in N^+ \\ j \neq i}} \tau_{i,j} x_{i,j} - \sum_{i \in N^-} \sum_{\substack{j \in N^+ \\ j \neq i}} \tau_{i,j} x^s_{i,j}$$

Define $x^s_{i,j}$

$$x^s_{i,j} \leq x_{i,j}, \forall i \in N^-, j \in N^+, i \neq j$$

$$\sum_{\substack{j \in N^+ \\ j \neq i \\ i \neq c+1}} x^s_{i,j} - \sum_{\substack{j \in N^- \\ j \neq i \\ i \neq 0}} x^s_{j,i} = \begin{cases} 1, & \text{if } i = 0 \\ -1, & \text{if } i = s \\ 0, & \text{otherwise.} \end{cases}$$

$$\sum_{v \in V} y^v_i = 1 - z_i, \forall i \in C''$$

$$W \geq \sum_{i \in C''} (\tau'_{s,i} + \tau'_{i,s}) y^v_i, \forall v \in V$$

$$0 \leq y^v_i \leq 1, \forall i \in C'', v \in V$$

Add drone routin[g] constraints to enh[ance the] BMP, where th[e ...] relaxed to continu[ous ...]

Replace a_s with $\sum_{i \in N^-} \sum_{\substack{j \in N^+ \\ j \neq i}} \tau_{i,j} x^s_{i,j}$

Benders subproblem and cut generation

$$(BSP)\ \min W_z$$

$$\text{s.t.} \sum_{v \in V} y_i^v = 1 - z_i', \ \forall i \in C''$$

$$W_z \geq \sum_{i \in C''} (\tau_{s,i}' + \tau_{i,s}') y_i^v, \ \forall v \in V$$

$$y_i^v \in \{0, 1\}, \ \forall i \in C'', v \in V$$

$$W_z \in R^+$$

- The BSP is equivalent to assigning jobs to parallel machines to minimize makespan.
  - **Global optimum for the BSP is not always necessary.**
- Utilizes the MULTIFIT algorithm by Coffman et al. (1978).
  - Simple and very fast.
  - Guarantees a performance ratio of 13/11.

Let W_z' represent the objective value of the (BSP) under given z'. The Benders optimality cut is

$$W \geq W_z' - W_z'\Big(\sum_{\substack{i \in C'' \\ z_i'=1}} (1 - z_i) + \sum_{\substack{i \in C'' \\ z_i'=0}} z_i\Big)$$

Theorem 1. *Equation (45) is a valid Benders optimality cut.*

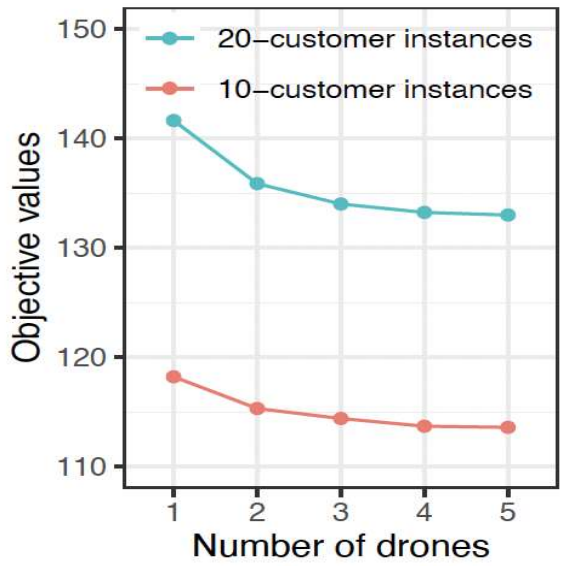
Algorithm 1 Pseudocode for adding Benders cuts to problem (BMP)

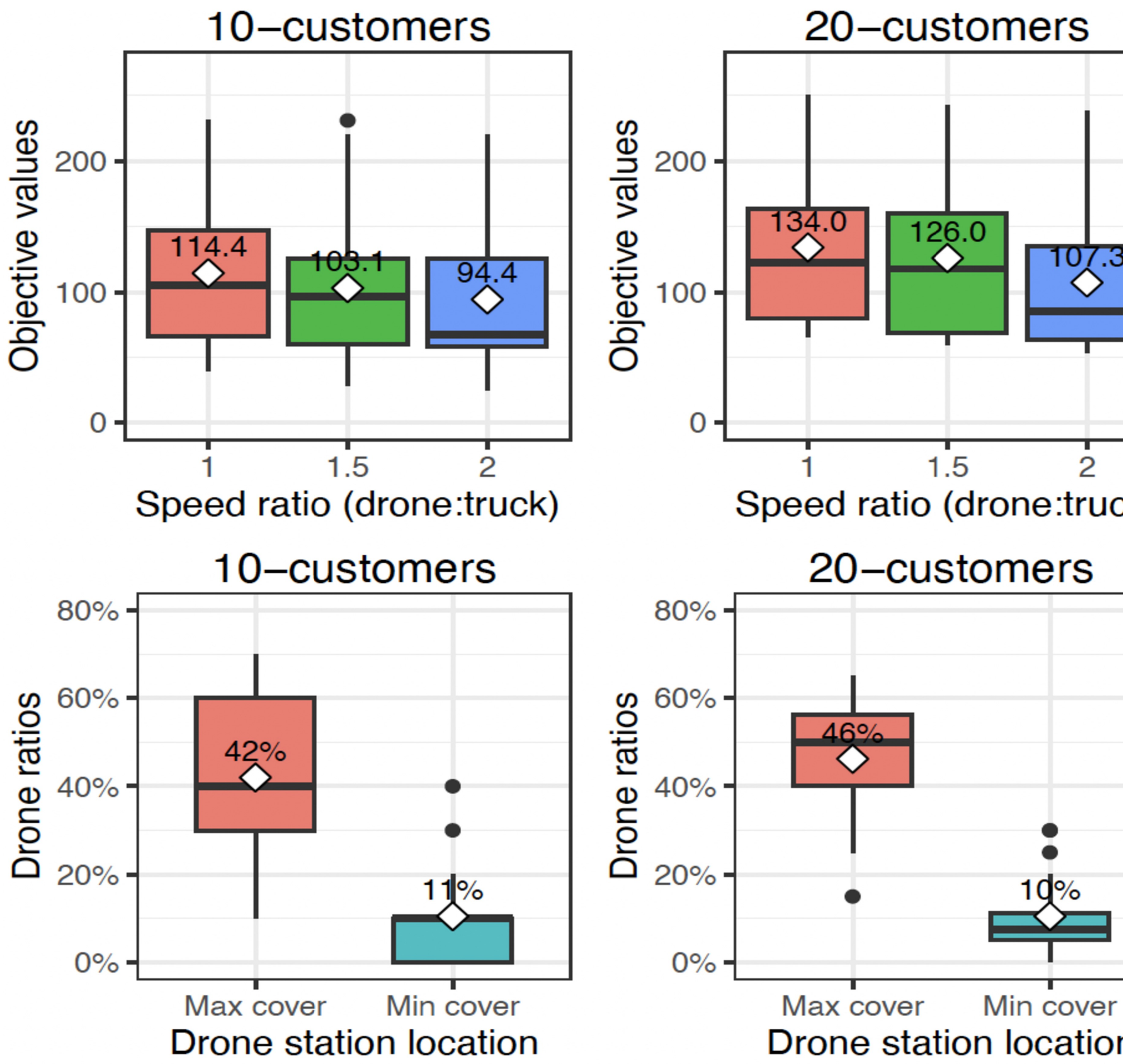
Input: Current solution of variable z in the (BMP): $\boldsymbol{z}^B$; Current soluti
W in the (BMP): W^B;

```
Solve the (BSP) with z^B;
if The objective value of the (BSP) is less than W^B then
    Terminate the solving procedure of the (BSP);
    Do not add Benders cuts to the (BMP);
else if Find the optimal solution of the (BSP) then
    if The optimal solution of the (BSP) equals to W^B then
        No Benders cut is found;
        Yield the optimal solution of the (P) from the current so
(BMP) and the optimal solution of the (BSP);
        Terminate the solving process, and return the optimal so
(P).
    else
        Add the Benders optimality cut to the (BMP);
    end if
end if
```

Some management insights

- Factors such as the number of drones, drone speed, and drone station location **significantly impact overall system performance**.
- Adding more drones yields **diminishing marginal gains**, highlighting the need to **balance costs and benefits**.





Conclusions and future directions

- **Conclusions：**
  - **Model Development**: An improved mixed-integer linear programming model for the Tra
    Salesman Problem with a Drone Station (TSP-DS) has been formulated.
  - **Algorithm Design**: A logic-based Benders decomposition algorithm was proposed based on the pr
    structure.
  - **Rigorous Testing**: The improved formulation and proposed algorithm were tested using ins
    generated from existing benchmarks.
  - **Management Insights**: Extensive sensitivity analyses provided management insights on how key
    parameters affect the performance of the delivery system.

- **Future Directions：**
  - **Exact Approaches**: Explore exact methods for scenarios involving **multiple trucks and drone sta**
    such as branch-price-cut techniques.
  - **Sensitivity Analysis**: Conduct sensitivity analyses considering the **interactions between**
    parameters (e.g., higher drone speed leads to shorter flight range due to increased energy consump
  - **Drone Station Location**: Investigate incorporating the **decision for drone station locations** or co
    replacing fixed drone stations with **mobile drone stations**, analogous to mobile depots.

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Thanks!

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